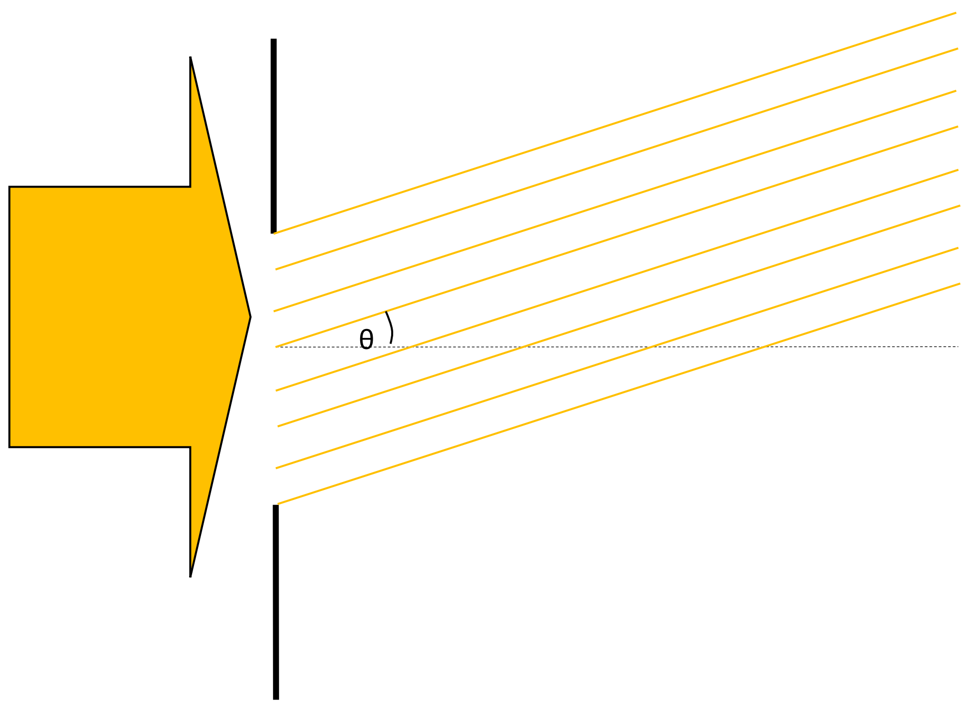
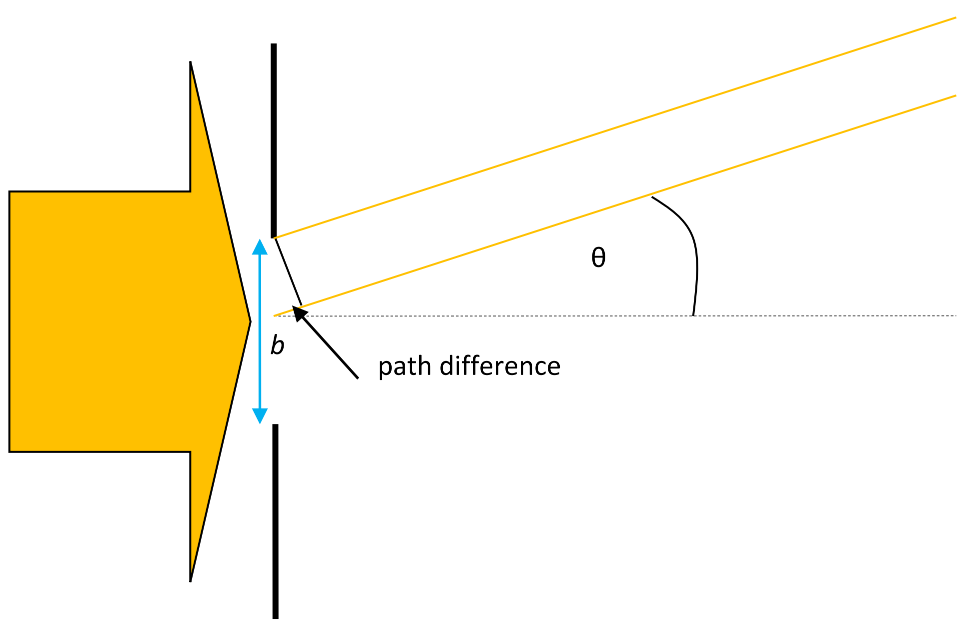
Teacher notes
Topic C

Single slit diffraction

Light diffracts going through the slit:

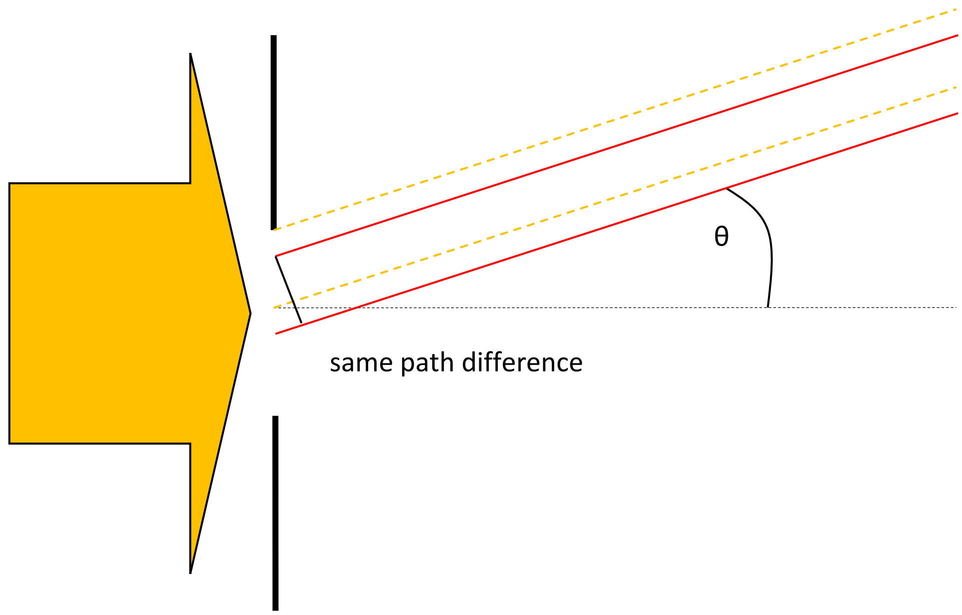


We concentrate on the rays leaving the top and middle of the slit:



The two rays shown have a path difference of $\frac{b}{2}\sin\theta \approx \frac{b}{2}\theta$, since θ is very small.

Now consider another pair of rays whose origins are just below the origins of the first 2:



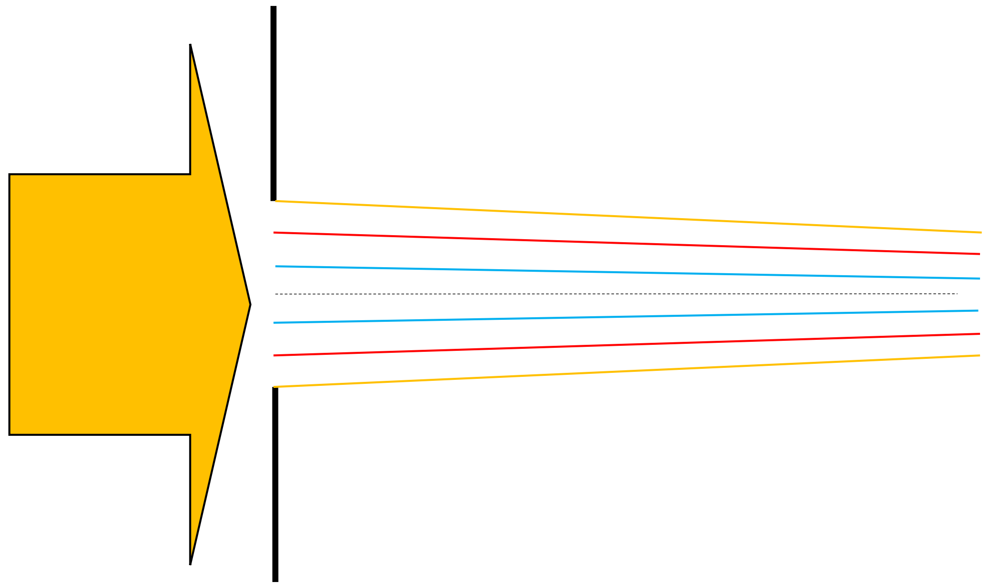
These have the same path difference. Considering similar pairs we see that if each pair has a path difference of $\frac{\lambda}{2}$ all pairs will result in destructive interference on the screen far away. The first minimum is thus obtained when

$$\frac{b}{2}\theta = \frac{\lambda}{2}$$

i.e. at

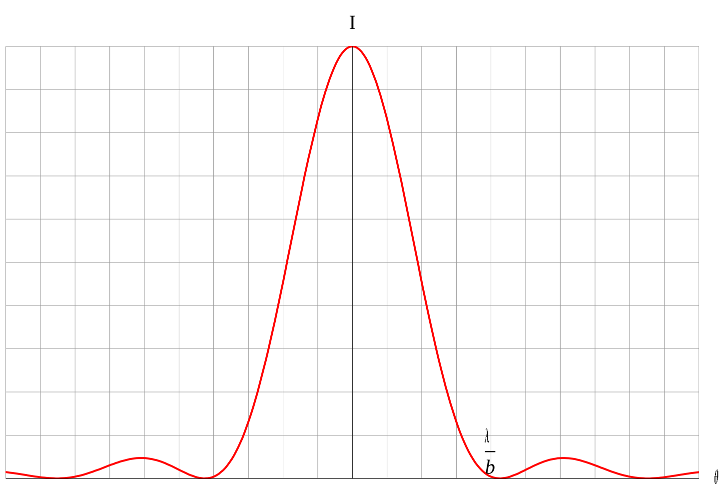
$$\theta = \frac{\lambda}{b}$$

The central maximum is obtained from rays that diffract as follows:



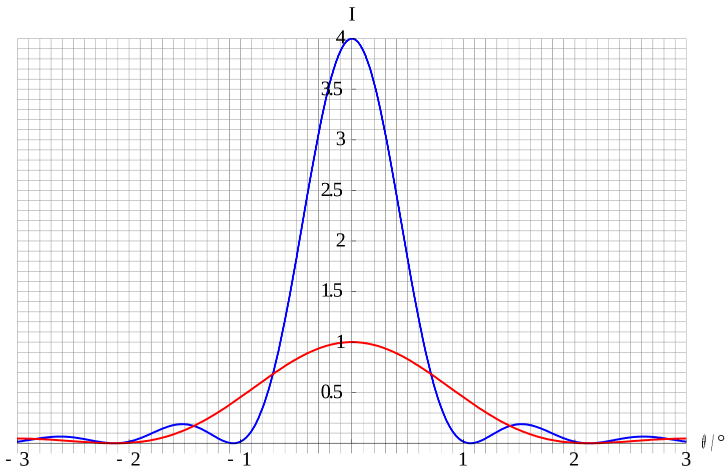
Each pair shown in the same colour has zero path difference so they all interfere constructively.

The one slit diffraction intensity pattern is given by this graph:



The first minimum occurs at $\theta = \dfrac{\lambda}{b}$ as shown above.

If the slit size b is doubled, the angle at which the first minimum appears is halved. Going to the very first diagram in this note we see that the number of rays through the slit will also double. Each ray contributes an amplitude A at the central maximum and so the total amplitude is NA. The intensity at the central point is proportional to the amplitude squared and so with double the slit width, N is doubled and so the intensity at the central point will increase by a factor of 4:



The blue curve corresponds to a slit width that is double that of the red curve. This is worth exploring. If we **double** the slit width, **double** the energy goes through. So why is the central maximum intensity 4 times as large? We can calculate the area under each curve which is proportional to the energy deposited on the screen. The blue curve is higher but it is also narrower. Doing the integrals, we find that the area under the blue curve is **double** that under the red curve *even though* the central peak of the blue curve is 4 times higher than the red peak. Conservation of energy holds. What is happening is what always happens in interference: energy is redistributed on the screen. Lots of energy near the central peak, less energy far away. (A curiosity: it turns out that the central peak contains about 90% of the total energy deposited on the screen.)

Notice that as the slit width *b* becomes larger and larger (compared to the wavelength), the angular position θ of the first minimum gets smaller and smaller: the diffraction pattern gets narrower and narrower. Notice also that if we do not use the small angle approximation the condition for the first minimum is $b\sin\theta = \lambda$. From this formula we can clearly see there is a lot of diffraction when $\lambda \geq b$: in this case $\sin\theta = \dfrac{\lambda}{b} \geq 1$ and so there is no angle at which the first minimum is observed. This means that the diffraction pattern has become so wide that we don't see any minima. In the other extreme case, $\lambda << b$, i.e. *b* large, $\sin\theta = \dfrac{\lambda}{b} \approx 0$. This means that the light is concentrated only at the position $\theta = 0$, i.e. the light goes straight through without being deviated.

In these graphs the intensity at the central maximum is normalized to 1.

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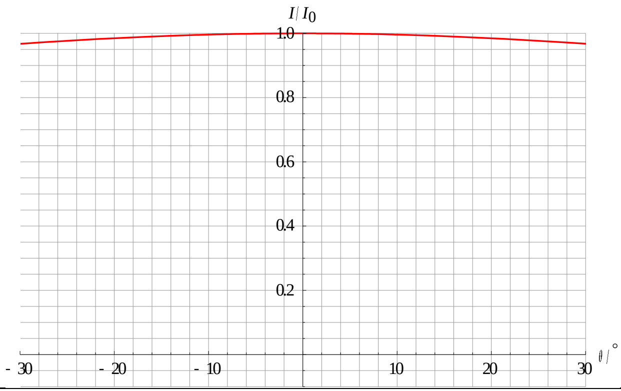
<td>
</td>
<td>$b=\dfrac{\lambda}{5}$ or $\lambda=5b$
The diffraction pattern is very wide. We have massive diffraction. No minima are observed. The wave has spread everywhere.</td>
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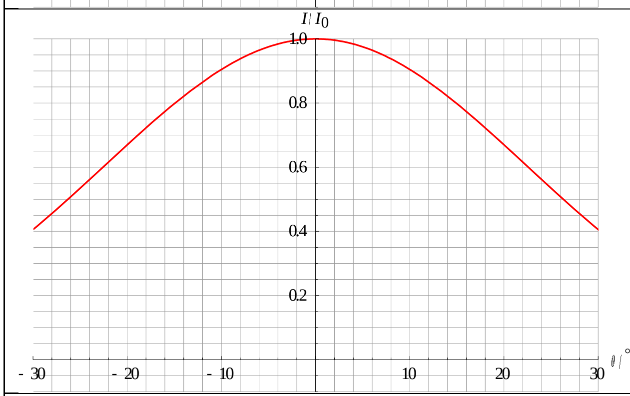
<td>
</td>
<td>$b\approx\lambda$
The diffraction pattern is very wide. We have massive diffraction. The wave has spread almost everywhere.</td>
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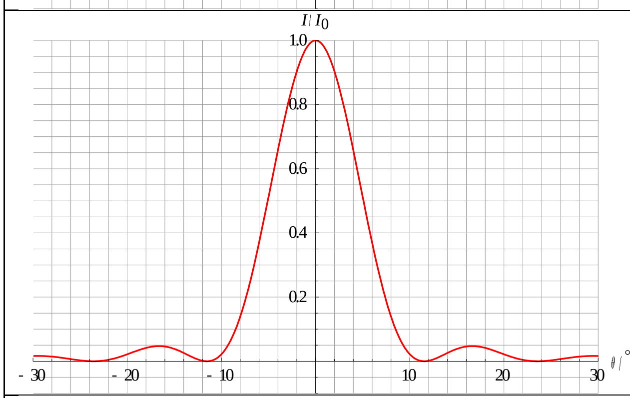
<td>
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<td>$b=5\lambda$ or $\lambda=\dfrac{b}{5}$
We have lots of diffraction including diffraction minima.</td>
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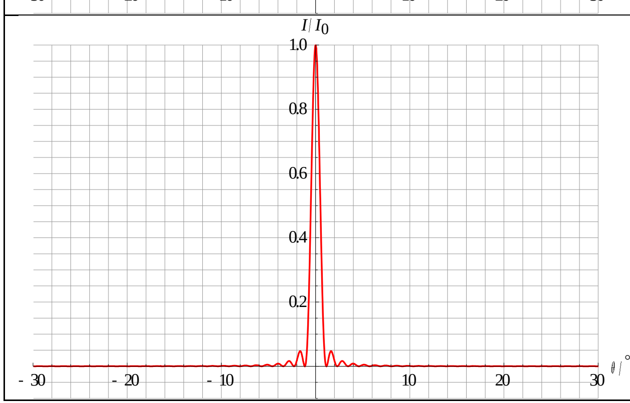
<td>
</td>
<td>$b=50\lambda$ or $\lambda=\dfrac{b}{50}$
The diffraction pattern is quite narrow. We have very limited diffraction. The light essentially goes straight through the slit without spreading.</td>
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